

Protected qubit based on a superconducting current mirror

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Abstract

We propose a qubit implementation based on exciton condensation in capacitively coupled Josephson junction chains. The qubit is protected in the sense that all unwanted terms in its effective Hamiltonian are exponentially suppressed as the chain length increases. We also describe an implementation of a universal set of quantum gates. Most gates also offer exponential error suppression. The only gate that is not intrinsically fault-tolerant needs to be realized with $\sim 50\%$ precision, provided the other gates are exact.

Physical implementation of a quantum computer presents a great challenge because quantum systems are susceptible to decoherence and because interactions between them cannot be controlled precisely. There has been impressive demonstrations of qubits using various kinds of systems, including Josephson junctions [1, 2, 3, 4, 5, 6], yet building a full-scale computer remains a remote goal. In principle, scalability can be achieved by correcting errors at the logical level [7], but only if the physical error rate is sufficiently small. As an alternative solution, it was suggested that topologically ordered quantum systems are physical analogues of quantum error-correcting codes, and fault-tolerant quantum computation can be performed by braiding anyons [8].

More recently, Doucot and Vidal [9], and Ioffe and Feigelman [10] found simpler examples of physical systems with error-correcting properties. The key element of their proposals, which we refer to as $0-\pi$ qubit, is a two-terminal circuit built of Josephson junctions. Its energy has two equal minima when the superconducting phase difference between the terminals, $\theta = \phi_1 - \phi_2$ is equal to 0 or π . The quantum states associated with the minima, $|0\rangle$ and $|1\rangle$ can form quantum superpositions. It is essential that the energy difference between the two minima is exponentially small in the system size, even in the presence of various perturbations, hence the quantum superposition will remain unchanged for a long time. Implementations of some quantum gates were also proposed.

In this paper, we discuss a different design of a $0-\pi$ qubit. It is based on the current mirror effect in capacitively coupled chains of Josephson junctions, see Fig. 1. An analogue of this effect in normal-metal junctions is due to correlated electron-hole tunneling [11], whereas in superconducting chains the tunneling objects are Cooper pairs. Positive and negative Cooper

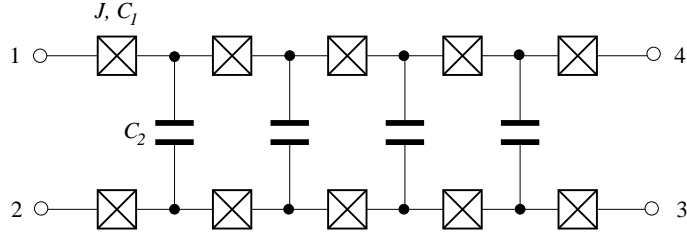


Figure 1: The superconducting current mirror.

pairs (with electric charge $+2e$ in one chain and $-2e$ in the other chain) tend to tunnel together. Under suitable conditions, the currents in the two chains are opposite in direction and almost equal in magnitude. This was observed experimentally [12] in the resistive state, i.e., at sufficiently large voltage bias. However, we will be concerned with a more delicate, dissipationless form of this effect, which has not been observed yet but predicted theoretically by Mahn-Soo Choi, M. Y. Choi, and Sung-Ik Lee [13] for the case of strong interchain coupling. In this regime, the Josephson junction ladder behaves as an almost perfect DC transformer with 1:1 current and voltage ratio.

Let each junction in the ladder have Josephson energy J and capacitance C_1 , and let C_2 be the interchain capacitance. If $C_2 \gg C_1$, excitons consisting of $+2e$ in one chain and $-2e$ in the other chain have lower energy than individual $\pm 2e$ quasiparticles or other excitations that change the total charge on some rungs of the ladder. The energy scales for excitons and unbalanced charges are given by $E_{\text{ex}} \sim e^2/C_2$ and $E_1 \sim e^2/C_1$, respectively. Excitons form a Bose condensate if $E_{\text{ex}} \lesssim J_{\text{ex}}$, where $J_{\text{ex}} \sim J^2/E_1$ is a characteristic hopping energy (we assume that $J \lesssim E_1$). In this regime, the system becomes superconducting with respect to opposite currents in the two chains while being insulating with respect to passing net electric charge along the ladder. It is worth noting that the exciton condensate persists in the presence of charge frustration [14].

The current mirror device may be characterized by an effective potential energy E that depends on the values of the superconducting phase at the four terminals, $\phi_1, \phi_2, \phi_3, \phi_4$. The order parameter of the exciton condensate may be represented as the superconducting phase difference between the chains, which is equal to $\theta_l = \phi_1 - \phi_2$ at the left end of the ladder and $\theta_r = \phi_4 - \phi_3$ at its right end. Thus we expect the energy to depend primarily on $\theta_l - \theta_r = \phi_1 - \phi_2 + \phi_3 - \phi_4$:

$$E = F(\phi_1 - \phi_2 + \phi_3 - \phi_4) + f(\phi_1 - \phi_4, \phi_2 - \phi_3), \quad (1)$$

where f is an “error term”. Since the current through the j -th terminal is proportional to $\partial E / \partial \phi_j$, the error term characterizes the net current through the ladder. Such current can only be carried by $\pm 2e$ quasiparticles tunneling through the insulator, but this process is suppressed by factor $\exp(-N/N_0)$, where N is the length (the number of junctions in each chain) and $N_0 \sim 1$. On the other hand, the F term in Eq.(1) is of the order of J_{ex}/N .

Now we will explain the design of the $0-\pi$ qubit, which is very simple. Let us connect the four leads diagonally, i.e., 1 with 3 and 2 with 4. Thus $\phi_1 = \phi_3$, $\phi_2 = \phi_4$, and $E \approx F(2(\phi_1 - \phi_2))$ with exponential precision. Since the function $F(\theta)$ has a minimum at $\theta = 0$, the energy of the qubit has two minima: at $\phi_1 - \phi_2 = 0$ and at $\phi_1 - \phi_2 = \pi$ (note that all the variables ϕ_j are defined modulo 2π). The energy values at the minima are exponentially close to each

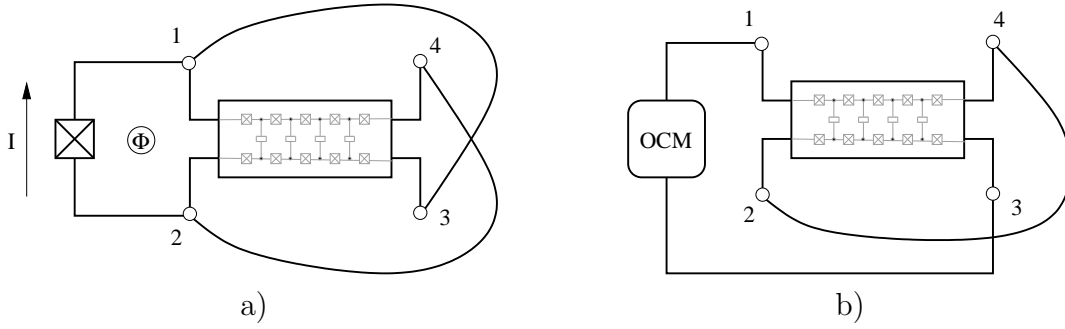


Figure 2: Measurement in the standard basis (a) and in the dual basis (b). Φ designates magnetic flux, and OCM a circuit for the offset charge measurement.

other: $\delta E \propto \exp(-N/N_0)$. That is the reason for protection against dephasing. To prevent bit flips, one needs to make sure that $E \gg e^2/C$, where $C = NC_2$ is total interchain capacitance. Note that the ratio $E/(e^2/C) \sim J_{\text{ex}}/E_{\text{ex}}$ does not depend on the length. It can be increased by increasing the interchain coupling or by connecting several current mirrors in parallel.

With this qubit design, it is possible to do measurements in the standard basis (of states $|0\rangle$ and $|1\rangle$ corresponding to $\phi_1 - \phi_2 = 0$ and $\phi_1 - \phi_2 = \pi$, respectively) as well as in the dual basis, $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. These may also be called “phase basis” and “charge basis”; the reason for the second name will be clear later. The measurement in the phase basis can be performed by simply connecting leads 1 and 2 to a measuring circuit. For example, if the leads are connected via a Josephson junction, the current in the loop depends on $\phi_1 - \phi_2 = 0, \pi$ and the magnetic flux through the loop, Fig. 2a.

The dual measurement is more complicated. The key idea is to break the wire between 1 and 3 and attach some measuring circuit to those leads, Fig. 2b. Now

$$\phi_2 = \phi_4 \equiv (\phi_1 + \phi_3)/2 \pmod{\pi}. \quad (2)$$

Furthermore, the potential energy is practically independent of the superconducting phase difference $\theta = \phi_1 - \phi_3$ across thus configured device, hence direct superconducting current cannot flow. The device behaves essentially as a capacitor with the effective Hamiltonian

$$H_C = \frac{(2e)^2}{2C} \left(\frac{\partial}{i \partial \theta} - n_g \right)^2, \quad (3)$$

except that it has an internal degree of freedom, since for fixed values of ϕ_1, ϕ_3 Eq. (2) has two solutions. The states $|+\rangle$ and $|-\rangle$ correspond to the symmetric and antisymmetric superposition of these solutions, and the wave function $\psi(\theta)$ satisfies the boundary condition $\psi(2\pi) = \psi(0)$ or $\psi(2\pi) = -\psi(0)$, respectively. The second boundary condition becomes equivalent to the first one if we change n_g by $1/2$. (The parameter n_g is a so-called offset charge measured in units of $2e$; it is defined modulo 1.) Thus, the measurement in the $|\pm\rangle$ basis amounts to distinguishing between n_g and $n_g + 1/2$. From the practical perspective, n_g need not be known in advance. Indeed, it is only important to tell the two states apart while the labels “+” and “−” can be assigned arbitrarily. We do not describe a concrete procedure for the offset charge measurement but only remark that this is a general problem not pertaining to the particular device.

The application of qubits for universal or specialized quantum computation requires some implementation of quantum gates. It is very desirable for the gates to be fault-tolerant at the

physical level, or at least to limit possible errors to some particular types. We now sketch such an implementation. It involves the choice of a nonstandard but computationally universal set of gates that are particularly suitable for the use with $0-\pi$ qubits. The computation is adaptive, i.e., it proceeds by intermediate measurements whose outcome determines the choice of the next gate to be applied. A tentative realization of the gates will be described briefly, with the intention to demonstrate the feasibility of the whole scheme.

The gates we use are as follows:

1. Measurement in the standard basis.
2. Measurement in the dual basis.
3. One-qubit unitary gate $R(\pi/4) = \exp(i(\pi/4)\sigma^z)$ and its inverse.
4. Two-qubit unitary gate $R_2(\pi/4) = \exp(i(\pi/4)\sigma_1^z\sigma_2^z)$ and its inverse.
5. One-qubit unitary gate $R(\pi/8) = \exp(i(\pi/8)\sigma^z)$ and its inverse.

To show that this set is universal, we first observe that repeated applications of noncommuting measurements allow one to prepare any of these states: $|0\rangle$, $|1\rangle$, $|+\rangle$, $|-\rangle$. Notice that $R(-\pi/4)$ is equal to the commonly used operator $\Lambda(i)$ up to an overall phase, where $\Lambda(i)|a\rangle = i^a|a\rangle$ for $a = 0, 1$. One can also implement the two-qubit controlled phase gate $\Lambda^2(-1)$ that acts as follows: $\Lambda^2(-1)|a, b\rangle = (-1)^{ab}|a, b\rangle$. Specifically, $\Lambda^2(-1)$ is equal to $R_2(\pi/4)\left(R(-\pi/4) \otimes R(-\pi/4)\right)$ up to an overall phase. If we add the Hadamard gate H , we obtain all Clifford (symplectic) gates. The Hadamard gate is realized by this adaptive procedure. We take a qubit in an arbitrary state $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$ and supplement it with a $|+\rangle$ ancilla. Then we apply $\Lambda^2(-1)$ and measure the first qubit in the dual basis. The second qubit now contains $H|\psi\rangle$ or $\sigma^x H|\psi\rangle$, depending on the measurement outcome. In the second case, we repeat the procedure 2, 4, 6, \dots more times until we achieve the desired result. Using the Clifford gates and the ability to create copies of $|\xi\rangle = R(\pi/8)|+\rangle$, one can perform quantum computation. Furthermore, if the Clifford gates are exact, the ancillary state $|\xi\rangle$ only needs to be prepared with fidelity $F > 0.93$ [15]. This gives more than 50% tolerance for choosing the parameter $u \approx \pi/8$ in $R(u) = \exp(iu\sigma^z)$.

The implementation of measurements is described above. One can readily see that it is fault-tolerant since the measured observable (i.e., the superconducting phase or the offset charge) is as unlikely to change during the process as in an isolated qubit. The gate $R(u)$ is realized by connecting the leads by a Josephson junction for a certain time period,¹ Fig. 3a. This procedure is generally sensitive to random variations of the time interval and the strength of the Josephson coupling.

A fault-tolerant implementation of $R(\pi/4) = \sqrt{i}\Lambda(-i)$ is based on a mathematical idea from Ref. [17]. The circuit is shown schematically in Fig. 3b: the $0-\pi$ qubit is connected to an ultraquantum LC -oscillator (with $r \stackrel{\text{def}}{=} (e^2/h)\sqrt{L/C} \gg 1$) for a certain time period τ . The operation of this gate may be described in terms of the superconducting phase difference θ across the inductor. It is a real variable (not identified modulo 2π) because there are no phase slips in the inductor. Initially, the oscillator is in its ground state characterized by a Gaussian

¹The required switch may be implemented as a series of SQUIDs controlled by magnetic field [16].

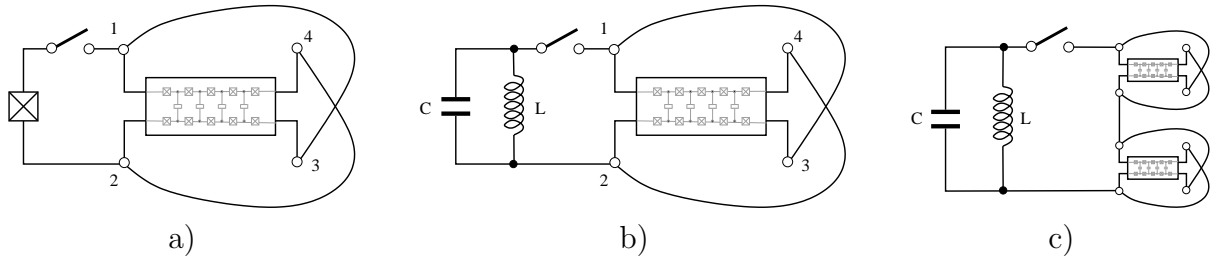


Figure 3: Realization of the gate $R(u)$ for arbitrary u (a); the fault-tolerant realization of $R(\pi/4)$ (b) and $R_2(\pi/4)$ (c).

wave function $\psi_0(\theta)$; note that $\langle \theta^2 \rangle \sim r \gg 1$. Once the circuit is closed, the quantum evolution is governed by the effective Hamiltonian

$$H_L = (\hbar^2/8e^2)L^{-1}\theta^2, \quad (4)$$

where θ takes on multiples of 2π if the qubit is in the state $|0\rangle$, or on values of the form $2\pi(n+1/2)$ if the qubit is in the state $|1\rangle$. Thus the wave function $\psi(\theta)$ has the form of a grid: it consists of narrow peaks at the said locations. If $\tau = 8L(e^2/h)$, then all peaks with $\theta = 2\pi n$ pick up no phase and all peaks with $\theta = 2\pi(n+1/2)$ are multiplied by $-i$. Thus the gate $\Lambda(-i)$ is effectively applied to the qubit state, not entangling it with the oscillator. Some conditions should be met for this scheme to work. The closing and breaking of the circuit should occur smoothly enough so that no excitation is produced in the switch itself, but faster than the LC oscillation period. The latter (antiadiabatic) condition is needed to prevent the squeezing of the state to just one or two peaks with the smallest energy. If these requirements are fulfilled, the qubit is transformed into a Gaussian grid state, i.e., a superposition of peaks with a broad Gaussian envelope. It is important that each peak is narrow and the Fourier image of the envelope is also narrow; such states are known to have good error-correcting properties [17]. Indeed, a bit flip is unlikely because the peaks from the $|0\rangle$ and $|1\rangle$ grids do not overlap, and dephasing is suppressed because $|+\rangle$ and $|-\rangle$ correspond to disjoint grids in the momentum space. If the protocol is not followed exactly but with a small error, it will mainly result in oscillations in the LC circuit after the cycle is complete, leaving the qubit unaffected. The gate $R_2(\pi/4)$ is implemented similarly: one just needs to connect the two $0-\pi$ qubits in series, Fig. 3c.

Summing up, we have described a concrete design that belongs to a class of $0-\pi$ superconducting qubits. It consists of a current mirror device with the four leads connected diagonally. This design makes it possible to perform a measurement with respect to the dual basis, using whatever technique that is suitable to measure the offset charge of a capacitor. We also propose a fault-tolerant scheme, including a universal gate set and their schematic implementations. This scheme can be used with any type of $0-\pi$ superconducting qubits.

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